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PAULI-VILLARS REGULARIZATION OF SUPERGRAVITY COUPLED TO CHIRAL AND YANG-MILLS MATTER*

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Abstract

It is shown that the one-loop quadratic divergences of standard supergravity can be regulated by the introduction of heavy Pauli-Villars fields belonging to chiral and abelian gauge multiplets. The resulting one-loop correction can be interpreted as a renormalization of the Kähler potential. Regularization of the dilaton couplings to the Yang-Mills sector requires special care, and may shed some light on chiral/linear multiplet duality of the dilaton supermultiplet.

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In extracting the phenomenological implications of an underlying supergravity theory, the quadratic divergences arising at one loop have often been considered [1]. However, the coefficients of the quadratically divergent terms are unreliable in the absence of a manifestly supersymmetric regularization procedure [2], [3]. The purpose of this Letter is to describe such a procedure.

The one-loop effective action S_1 is obtained from the term quadratic in quantum fields when the Lagrangian is expanded about an arbitrary background:

$$\begin{aligned}\mathcal{L}_{quad}(\Phi,\Theta,c) &= -\frac{1}{2}\Phi^T Z^\Phi\left(\hat{D}^2_\Phi + H_\Phi\right)\Phi + \frac{1}{2}\bar{\Theta}Z^\Theta\left(i\not{D}_\Theta - M_\Theta\right)\Theta \\ &\quad +\frac{1}{2}\bar{c}Z^c\left(\hat{D}^2_c + H_c\right)c + O(\psi), \end{aligned}\tag{1}$$

where the column vectors Φ, Θ, c represent quantum bosons, fermions and ghost fields, respectively, and ψ represents background fermions that we shall set to zero throughout this paper. The fermion sector Θ includes a C-odd Majorana auxiliary field α that is introduced to implement the gravitino gauge fixing condition. The full gauge fixing procedure used here is described in detail in [4], [5]. Then the one loop bosonic action is given by

$$\begin{aligned}S_1 &= \frac{i}{2}\mathrm{Tr}\ln\left(\hat{D}^2_\Phi + H_\Phi\right) - \frac{i}{2}\mathrm{Tr}\ln\left(-i\not{D}_\Theta + M_\Theta\right) + \frac{i}{2}\mathrm{STr}\ln\left(\hat{D}^2_c + H_c\right) \\ &= \frac{i}{2}\mathrm{STr}\ln\left(\hat{D}^2 + H\right) + T_-, \end{aligned}\tag{2}$$

where T_- is the helicity-odd fermion contribution which contains no quadratic divergences, and the helicity-even contribution is determined by

$$\hat{D}^2_\Theta + H_\Theta \equiv \left(-i\not{D}_\Theta + M_\Theta\right)\left(i\not{D}_\Theta + M_\Theta\right). \tag{3}$$

The field-dependent matrices $H(\phi)$ and $\hat{D}_\mu(\phi) = \partial_\mu + \Gamma_\mu(\phi)$ are given in [4], [5], where the logarithmically divergent contributions have been evaluated. Explicitly evaluating (2) with an ultraviolet cut-off Λ and a massive Pauli-Villars sector with a squared mass matrix of the form

$$M^2_{PV} = H^{PV}(\phi) + \begin{pmatrix}\mu^2 & \nu \\ \nu^\dagger & \mu^2\end{pmatrix} \equiv H^{PV} + \mu^2 + \nu, \quad |\nu|^2 \sim \mu^2 \gg H^{PV} \sim H,$$

gives, with $H' = H + H^{PV}$:

$$\begin{aligned}32\pi^2 S_1 &= -\int d^4x d^4p \mathrm{STr} \ln\left(p^2+\mu^2+H'+\nu\right) + 32\pi^2\left(S_1'+T_-\right)\\ &= 32\pi^2\left(S_1'+T_-\right) - \int d^4x d^4p \mathrm{STr}\ln\left(p^2+\mu^2\right)\\ &\quad -\int d^4x d^4p \mathrm{STr}\ln\left[1+\left(p^2+\mu^2\right)^{-1}\left(H'+\nu\right)\right]. \end{aligned} \tag{4}$$

S_1' is a logarithmically divergent contribution that involves the operator $\hat{G}_{\mu\nu} = [\hat{D}_\mu, \hat{D}_\nu]$. Finiteness of (4) requires

$$\mathrm{STr}\mu^{2n} = \mathrm{STr}H' = \mathrm{STr}\left(2\mu^2 H' + \nu^2\right) = 0, \quad \frac{1}{64\pi^2}\mathrm{STr}H'^2 = -L, \tag{5}$$

where L is the coefficient of $\ln\Lambda^2$ in $S_1' + T_-$. The vanishing of $\mathrm{STr}\mu^{2n}$ is automatically assured by supersymmetry. Once the remaining conditions are satisfied we obtain

$$S_1 = -\int \frac{d^4x}{64\pi^2}\mathrm{STr}\left[\left(2\mu^2 H' + \nu^2\right)\ln\mu^2\right] + O(\ln\mu^2). \tag{6}$$

First consider a supergravity theory in which the Yang-Mills fields have canonical kinetic energy. Then the quadratically divergent contributions from the (gauge-fixed) gravity sector, the N chiral supermultiplets and the (gauge-fixed) Yang-Mills sector of internal symmetry dimension N_G, are respectively:

$$\begin{aligned}\mathrm{STr}H_{grav} &= -10V - 2M_\psi^2 - \frac{r}{2} + 4K_{i\bar{m}}\mathcal{D}_\nu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} - \frac{x}{2}F^2,\\ \mathrm{STr}H_\chi &= 2N\left(\hat{V} + M_\psi^2 - \frac{r}{4}\right) + 2x^{-1}\mathcal{D}_a D_i (T^a z)^i\\ &\quad -2R_{i\bar{m}}\left(e^{-K}\bar{A}^i A^{\bar{m}} + \mathcal{D}_\nu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}}\right),\\ \mathrm{STr}H_{YM} &= 2\mathcal{D} + \frac{x}{2}F^2 + N_G\frac{r}{2}. \end{aligned} \tag{7}$$

In these expressions, r is the space-time curvature, $F^2 = F^a_{\mu\nu}F_a^{\mu\nu}$ with $F^a_{\mu\nu}$ the Yang-Mills field strength, $x = g^{-2}$ is the inverse squared gauge coupling constant, $K_{i\bar{m}}$ is the Kähler metric and $R_{i\bar{m}}$ is the associated Ricci tensor, $V = \hat{V} + \mathcal{D}$ is the classical scalar potential with $\hat{V} = e^{-K}A_i\bar{A}^i - 3M_\psi^2$, $\mathcal{D} =$

$(2x)^{-1}\mathcal{D}^a\mathcal{D}_a$, $\mathcal{D}_a = K_i(T_az)^i$, and $M^2_\psi = e^{-K}A\bar{A}$ is the field-dependent squared gravitino mass, with

$$A = e^K W = \bar{A}^\dagger, \quad A_i = D_iA, \quad \bar{A}^i = K^{i\bar{m}}\bar{A}_{\bar{m}}, \quad etc., \tag{8}$$

where D_i is the scalar field reparametrization covariant derivative. The F^2 terms in (7) cancel in the overall supertrace. To regulate the z-dependent terms, where $z = \bar{z}^\dagger$ is the scalar superpartner of a left-handed fermion, we introduce Pauli-Villars regulator chiral supermultiplets $Z^I_\alpha = (\bar{Z}^{\bar{I}}_\alpha)^\dagger$, $Z'^I_\alpha = (\bar{Z}'^{\bar{I}}_\alpha)^\dagger$ and $\varphi^A = (\bar{\varphi}^A)^\dagger$, with Kähler potential:

$$K(Z,\bar{Z},\varphi,\bar{\varphi}) = \sum_{\alpha,I=i,M=m} K_{i\bar{m}}(z,\bar{z})\left(Z^I_\alpha\bar{Z}^{\bar{M}}_\alpha + Z'^I_\alpha\bar{Z}'^{\bar{M}}_\alpha\right) + \sum_A e^{\alpha_A K}\varphi^A\bar{\varphi}^A, \tag{9}$$

superpotential:

$$W(Z,\varphi) = \sum_{A,I}\mu^\alpha_I Z^I_\alpha Z'^I_\alpha + \sum_A \mu_A\left(\varphi^A\right)^2, \tag{10}$$

and signature $\eta^{\alpha,A} = \pm 1$, which determines the sign of the corresponding contribution to the supertrace relative to an ordinary particle of the same spin. Thus $\eta = +1(-1)$ for ordinary particles (ghosts). The contribution of Pauli-Villars loops should be regarded as a parametrization of the result of integrating out heavy (*e.g.* Kaluza-Klein or string) modes of an underlying finite theory; the contributions from Pauli-Villars fields with negative signature could be interpreted as those of ghosts corresponding to heavy fields of higher spin. Z^I transforms like z^i under the gauge group, and Z'^I transforms according to the conjugate representation.

In evaluating the effective one-loop action we set to zero all background Pauli-Villars fields; then the contribution of these fields to $\mathrm{STr}H_\chi$ is

$$\begin{aligned}\mathrm{STr}H^{PV}_\chi \;=\;& 2\sum_{\alpha,A}(2\eta_\alpha+\eta_A)\left(\hat{V}+M^2_\psi-\frac{r}{4}\right)\\ &+4\sum_{\alpha,J}\eta_\alpha\left[x^{-1}\mathcal{D}^aD_{(\alpha J)}(T_az)^{(\alpha J)} - R^{(\alpha J)}_{(\alpha J)i\bar{m}}\left(\bar{A}^iA^{\bar{m}}e^{-K}+\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}\right)\right]\\ &+2\sum_A\eta_A\left[x^{-1}\mathcal{D}^aD_A(T_az)^A - R^A_{Ai\bar{m}}\left(\bar{A}^iA^{\bar{m}}e^{-K}+\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}\right)\right].\end{aligned} \tag{11}$$

From (9) we obtain for the relevant elements of the scalar reparametrization connection Γ and Riemann tensor R:

$$\begin{aligned}\Gamma^A_{Bk} &= \alpha_A\delta^A_B K_k \quad R^A_{Bk\bar m} = \alpha_A\delta^A_B K_{k\bar m}, \quad D_A(T_az)^A = \alpha_A K_i(T_az)^i,\\ \Gamma^{(I\alpha)}_{(J\beta),k} &= \delta^\alpha_\beta\Gamma^i_{jk}, \quad R^{(I\alpha)}_{(J\beta)k\bar m} = \delta^\alpha_\beta\delta^I_J R^i_{jk\bar m}, \quad D_I(T_az)^I = D_i(T_az)^i. \qquad (12)\end{aligned}$$

Finally, to regulate the r-dependent term in $\mathrm{STr}H$ we introduce $U(1)$ gauge supermultiplets W^a with signature η^a and chiral multiplets $Z^a = e^{\theta^a} = (Z^a)^\dagger = \left(e^{\bar\theta^a}\right)^\dagger$ with the same signature, $U(1)_b$ charge $q_a\delta_{ab}$, and Kähler potential:

$$K(\theta,\bar\theta) = \frac{1}{2}\sum_a \nu_a e^{\alpha_a K}(\theta_a + \bar\theta_a)^2 \qquad (13)$$

which is invariant under $U(1)_b$: $\delta_b\theta_a = -\delta_b\bar\theta_a = iq_a\delta_{ab}$. The corresponding D-term:

$$\mathcal{D}(\theta,\bar\theta) = \frac{1}{2x}\mathcal{D}^a_\theta\mathcal{D}^\theta_a, \quad \mathcal{D}^\theta_a = \sum_b K_b\delta_a\theta^b = i(\theta^a + \bar\theta^a)q_a e^{\alpha_a K}\nu_a, \qquad (14)$$

vanishes in the background, but $(\theta^a + \bar\theta^a)/\sqrt{2}$ acquires a squared mass $\mu_a^2 = (2x)^{-1}q_a^2 e^{\alpha_a K}\nu_a$ equal to that of W^a, with which it forms a massive vector supermultiplet. The chiral multiplets contribute to $\mathrm{STr}H^{PV}$ in the same way as φ^A with $\alpha_A \to \alpha_a$, and the vector multiplets contribute to the r-term with opposite sign. Therefore we obtain an overall contribution from light and heavy modes:

$$\begin{aligned}\mathrm{STr}H' &= 2\hat V\left[N\left(1 + 2\sum_\alpha\eta_\alpha\right) + \sum_A\eta_A\left(1-\alpha_A\right) + \sum_a\eta_a\left(1-\alpha_a\right) - 5\right]\\ &\quad +2M_\psi^2\left[N\left(1 + 2\sum_\alpha\eta_\alpha\right) + \sum_A\eta_A\left(1-3\alpha_A\right) + \sum_a\eta_a\left(1-3\alpha_a\right) - 1\right]\\ &\quad -\frac{r}{2}\left[N\left(1 + 2\sum_\alpha\eta_\alpha\right) + \sum_A\eta_A + 1 - N_G\right]\\ &\quad +2\left[x^{-1}\mathcal{D}_a D_i(T^az)^i - R_{i\bar m}\left(e^{-K}\bar A^i A^{\bar m} + \mathcal{D}_\nu z^i\mathcal{D}^\mu\bar z^{\bar m}\right)\right]\left(1 + 2\sum_\alpha\eta_\alpha\right)\\ &\quad +2\left(K_{i\bar m}\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar z^{\bar m} - 2\mathcal{D}\right)\left(2 - \sum_A\eta_A\alpha_A - \sum_a\eta_a\alpha_a\right). \qquad (15)\end{aligned}$$

Thus $\mathrm{STr}H' = 0$ requires

$$
\begin{aligned}
0 &= 1 + 2\sum_\alpha \eta_\alpha = \sum_A \eta_A + \sum_a \eta_a - 7 \\
&= \sum_A \eta_A + 1 - N_G = 2 - \sum_A \eta_A \alpha_A - \sum_a \eta_a \alpha_a. \qquad (16)
\end{aligned}
$$

The vanishing of $\mathrm{STr}(\mu^2 H' + \nu^2)$ further constrains the parameters $\mu(z)$ and $\nu(z,\bar{z})$, which can in general depend on the light chiral multiplets. For example, if the underlying theory is a superstring theory, there is usually invariance under a modular transformation on the light superfields under which $K \to K + F(z) + \bar{F}(\bar{z}),\ \ W \to e^{-F(z)}W$, which cannot be broken by perturbative quantum corrections [6]. Thus the field Z^I_α has the same modular weight as z^i, φ^A has modular weight $-\alpha_A/2$; the z-dependence of $\mu(z)$, as well as of $\nu_a(z,\bar{z})$ must be chosen so as to restore modular invariance. The result would be interpreted as threshold effects arising from the integration over heavy modes. (It is possible that some or all of the modular invariance may be restored by a universal Green-Schwarz type counter term, as is the case for the anomalous Yang-Mills coupling [7]–[10].) In the following we set $q_a = 1,\ \mu^\alpha_I = \beta^Z_\alpha \mu_I(z),\ \mu_A = \beta_A \mu_\varphi(z),\ \nu_a = x\beta^2_a |\mu_\theta(z)|^2$, with $\beta_{\alpha,A,a}$ independent of z and $\alpha_{A,a} \equiv \alpha_{\varphi,\theta}$ independent of A, a. Since $\mathrm{STr}(2\mu^2 H' + \nu^2)$ is just the $O(\mu^2)$ part of $\mathrm{STr}(\mu^2 + H' + \nu)^2$, it can be read off from the general results of [4],[5], with $H \to H' + \mu^2 + \nu \equiv \tilde{H}$. The terms in $\frac{1}{2}\mathrm{STr}\tilde{H}$ proportional to μ^2 are:

$$
\begin{aligned}
\frac{1}{2}\mathrm{STr}\tilde{H} \ \ni\ & e^{-K} A_{IJ}\bar{A}^{IJ}\left[\left(2\hat{V} + 3M^2_\psi\right) - \frac{r}{2}\right] + 2e^{-2K} A_{IJ}\bar{A}^{JK} R^{m\ I}_{\ n\ K} A_m \bar{A}^n \\
& + e^{-2K}\left[A_{kIJ}\bar{A}^{IJm}\bar{A}^k A_m - \left(A_{IjK}\bar{A}^{IK}\bar{A}^j A + \mathrm{h.c.}\right)\right] \\
& + \mathcal{D}_\mu \bar{z}^{\bar{m}} \mathcal{D}^\mu z^i e^{-K}\left(A_{iJK}\bar{A}^{JK}_{\bar{m}} + 2R^K_{i\bar{m}J} e^{-K} A_{KL}\bar{A}^{JL}\right) \\
& - \frac{e^{-K}}{x}\mathcal{D}_a (T^a z)^i A_{iJK}\bar{A}^{JK} \\
& - \frac{4e^{-K}}{x}\delta^a\theta^c \delta_a \theta^{\bar{b}}\left[A_{dc}\bar{A}^d_{\bar{b}} - R^k_{\ n\bar{b}c} A_k \bar{A}^n\right] + 4\left(\hat{V} + M^2_\psi\right)\mathcal{K}^a_a \\
& + \frac{4}{x}\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}}\left[R_{\bar{m}i\bar{b}c}\delta_a\theta^c \delta^a \theta^{\bar{b}} - K_{c\bar{b}} D_{\bar{m}} \delta_a \theta^{\bar{b}} D_j \delta^a \theta^c\right]. \qquad (17)
\end{aligned}
$$

where here upper case indices refer to Z^I_α, Z'^I_α and φ^A. Lower indices denote scalar field reparametrization invariant derivatives, and indices are raised with the inverse metric. The relevant matrix elements are, in addition to (12):

$$\begin{aligned}
\sum_J e^{-K} A_{I\alpha,J\alpha}\bar{A}^{K\alpha,J\alpha} &= \delta^K_I \sum_{m=M} e^K \left(K^{i\bar{m}}\beta^Z_\alpha\right)^2 \mu_I \bar{\mu}_{\bar{M}} \equiv \delta^K_I \left(\beta^Z_\alpha\right)^2 \Lambda^2_I \\
A_{I\alpha,J\alpha,k} &= \left(K_k - \partial_k \ln \mu_I\right) A_{I\alpha,J\alpha} - 2\Gamma^\ell_{ki} A_{L\alpha,J\alpha} \\
e^{-K} A_{CB}\bar{A}^{AC} &= \delta^A_B e^{K(1-2\alpha_\varphi)} |\beta_A \mu_\varphi|^2 \equiv \delta^A_B \beta^2_A \Lambda^2_\varphi \\
A_{ABi} &= \left[K_i(1-2\alpha_A) - \partial_i \ln \mu_\varphi\right] A_{AB}
\end{aligned}$$

$$\begin{aligned}
\mathcal{K}^a_a &= \frac{1}{x}\sum_{b,c} \delta^a\theta^c K_{c\bar{b}} \delta_a \bar{\theta}^{\bar{b}} = |\beta_a \mu_\theta|^2 e^{\alpha_\theta K} \equiv \beta^2_a \Lambda^2_\theta, \\
A_{bc} &= \nu_b e^{\alpha_\theta K} A^{\bar{b}}_c = K_{ab}A - \Gamma^i_{ab} A_i = \nu_b e^{\alpha_\theta K}\left[A - \left(\alpha_\theta K_{\bar{m}} + 2\partial_m \ln \bar{\mu}_\theta\right) A^{\bar{m}}\right], \\
D_i \delta_a \theta^c &= \Gamma^c_{ib} \delta_a \theta^b = \left(\alpha_\theta K_i + 2\partial_i \ln \mu_\theta\right) \delta_a \theta^c,
\end{aligned}$$

$$\sum_a \delta^a\theta^c \delta_a \theta^{\bar{b}} R_{i\bar{m}c\bar{b}} = -\delta^c_b \beta^2_b \alpha_\theta K_{i\bar{m}} \Lambda^2_\theta \tag{18}$$

The finiteness constraint requires

$$\sum_\alpha \eta^Z_\alpha \left(\beta^Z_\alpha\right)^2 = \sum_A \eta_A \left(\beta_A\right)^2 = \sum_a \eta_a \left(\beta_a\right)^2 = 0. \tag{19}$$

Then the results of [4], [5] determine the $O(\mu^2)$ contribution to $S_0 + S_1 = \int d^4x \left(\mathcal{L}_0 + \mathcal{L}_1\right)$:

$$\mathcal{L}_0(g^0_{\mu\nu}, K) + \mathcal{L}_1 = \mathcal{L}_0(g_{\mu\nu}, K + \delta K), \quad g_{\mu\nu} = g^0_{\mu\nu}\left(1+\epsilon\right)$$

$$\begin{aligned}
\epsilon &= -\sum_P \frac{\lambda_P}{32\pi^2} e^{-K} A_{PQ}\bar{A}^{PQ} = \sum_\Phi \frac{\lambda_\Phi \Lambda^2_\Phi}{32\pi^2}\zeta'_\Phi, \\
\delta K &= \sum_P \frac{\lambda_P}{32\pi^2}\left(e^{-K} A_{PQ}\bar{A}^{PQ} - 4\mathcal{K}^P_P\right) = \sum_\Phi \frac{\lambda_\Phi \Lambda^2_\Phi}{32\pi^2}\zeta_\Phi,
\end{aligned} \tag{20}$$

where [11] $\lambda_\Phi = 2\sum_p \eta^\Phi_p \left(\beta^\Phi_p\right)^2 \ln \beta^p_\Phi$, $\zeta_Z = \zeta_\varphi = \zeta'_Z = \zeta'_\varphi = 1$, $\zeta_\theta = -4$ $\zeta'_\theta = 0$, and P, Q denote all heavy modes. It should be emphasized that if there are

three or more terms in the sum over p, the sign of λ_Φ is indeterminate [11], so caution should be used in making conclusions about the implications of these terms for the stability of the effective potential.

Before proceeding to the case of noncanonical gauge field kinetic energy, we note that there is an ambiguity in the separation of the fermion loop contribution into helicity-odd and -even parts. We define [5]:

$$-\frac{i}{2}\mathrm{Tr}\ln(-i\not{D}+M_\Theta)\equiv-\frac{i}{2}\mathrm{Tr}\ln\mathcal{M}(\gamma_5)=T_-+T_+,$$

$$\begin{aligned}
T_- &= -\frac{i}{4}\left[\mathrm{Tr}\ln\mathcal{M}(\gamma_5)-\mathrm{Tr}\ln\mathcal{M}(-\gamma_5)\right],\\
T_+ &= -\frac{i}{4}\left[\mathrm{Tr}\ln\mathcal{M}(\gamma_5)+\mathrm{Tr}\ln\mathcal{M}(-\gamma_5)\right],\\
\mathcal{M} &= \gamma_0(-i\not{D}+M_\Theta)=\begin{pmatrix}\sigma^\mu_+D^+_\mu & M^+\\ M^- & \sigma^\mu_-D^-_\mu\end{pmatrix},\qquad \sigma^\mu_\pm=(1,\pm\vec{\sigma}).
\end{aligned}\tag{21}$$

Thus if $D_\mu=\partial_\mu+V_\mu+iA_\mu\gamma_5,\ M=m+m'\gamma_5$, then $D^\pm_\mu=\partial_\mu+V_\mu\pm iA_\mu,\ M^\pm=m\mp m'$. The ambiguity arises because we can interchange terms that are even and odd in γ_5 using $\gamma_5=(i/24)\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma$ and similar identities. In most cases the correct choice is dictated by gauge or Kähler covariance. However there is an off-diagonal mass term that mixes gauginos with the auxiliary field α:

$$M_{\alpha\lambda^a}=-\sqrt{\frac{x}{2}}F^{\mu\nu}_a\sigma_{\mu\nu}=-\sqrt{\frac{x}{2}}\left(\alpha F^{\mu\nu}_a+i\beta\gamma_5\tilde{F}^{\mu\nu}_a\right)\sigma_{\mu\nu},\qquad \alpha+\beta=1.\tag{22}$$

The result is invariant under the choice of α only if the integrals are finite. In the above we took $\alpha=1,\ \beta=0$; with an arbitrary choice we would have gotten, instead of (7):

$$\begin{aligned}
\mathrm{STr}H_{grav} &= -10V-2M^2_\psi-\frac{r}{2}+4K_{i\bar{m}}\mathcal{D}_\nu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}+\frac{x}{2}F^2(\alpha^2-\beta^2-2),\\
\mathrm{STr}H_{YM} &= 2\mathcal{D}+\frac{x}{2}F^2(\alpha^2-\beta^2)+N_G\frac{r}{2}.
\end{aligned}\tag{23}$$

The choice used above is “supersymmetric” in the sense that it corresponds to analogous matrix elements [5] in the bosonic and ghost sectors, yielding the cancellation of the F^2 terms.

Now we introduce the dilaton; that is, we couple the Yang-Mills sector to a holomorphic function of the chiral multiplets: $f_{ab} = \delta_{ab} f, \quad f = x + iy$. (The results can immediately be generalized to the case $f_{ab} = \delta_{ab} k_a f, \ k_a =$ constant, by making the substitutions $F^a_{\mu\nu} \to k_a^{\frac{1}{2}} F^a_{\mu\nu}, \ A^a_\mu \to k_a^{\frac{1}{2}} A^a_\mu, \ T^a \to k_a^{-\frac{1}{2}} T^a$.) There is a dilatino-gaugino mass term and an additional gaugino connection that can be written as

$$\begin{aligned} M_{\chi^i \lambda^a} &= -i \frac{f_i}{4\sqrt{x}} \left(\gamma F_a^{\mu\nu} + i\delta\gamma_5 \tilde{F}_a^{\mu\nu} \right) \sigma_{\mu\nu}, \quad \gamma + \delta = 1, \quad f_i = \partial_i f \\ A^\mu_{\lambda^a \lambda^b} &= -\delta_{ab} \frac{\partial^\mu y}{2x} \left(i\epsilon\gamma_5 - \zeta \frac{\epsilon^{\lambda\nu\rho\sigma}}{24} \gamma_\lambda \gamma_\nu \gamma_\rho \gamma_\sigma \right), \quad \epsilon + \zeta = 1. \end{aligned} \tag{24}$$

Then we obtain the additional contributions to the supertraces:

$$\begin{aligned} \mathrm{STr} H_{YM} &\ni \frac{f_i \bar{f}^i}{4x} F^2 (\gamma^2 - \delta^2) - N_G \left(2M_\lambda^2 + \frac{1}{2x^2} \left[\partial_\mu x \partial^\mu x + (3 - 2\zeta^2) \partial_\mu y \partial^\mu y \right] \right), \\ \mathrm{STr} H_{grav} &\ni \frac{f_i \bar{f}^i}{4x} F^2 (\gamma^2 - \delta^2) - \frac{f_i \bar{f}^i}{2x^2} \mathcal{D}, \quad \mathrm{STr} H_\chi \ni \frac{f_i \bar{f}^i}{2x^2} \mathcal{D}, \end{aligned} \tag{25}$$

where $M_\lambda^2 = (2x)^{-2} e^{-K} f_i \bar{f}^j A_j \bar{A}^i, \ \bar{f}^i = K^{i\bar{m}} \bar{f}_{\bar{m}}$. The "supersymmetric" choice, which matches corresponding matrix elements [5] in the bosonic and ghost sectors, is $\gamma = \delta = \frac{1}{2}, \ \epsilon = 0, \ \zeta = 1$. Then the F^2 terms again cancel, and the remaining terms:

$$\mathrm{STr} H \ni -N_G \left(2M_\lambda^2 + \frac{1}{2x^2} \left[\partial_\mu x \partial^\mu x + \partial_\mu y \partial^\mu y \right] \right), \tag{26}$$

can be regulated by the introduction of additional Pauli-Villars chiral multiplets, as will be shown below. With any other choice cancellation of the infinities would be achieved only through the introduction of Pauli-Villars "dilatons" and/or "gauge fields" with linear couplings to the light, physical fields, thus entailing loops mixing quantum fields of different signature. With the choice $\zeta = 1$, the y-axion contribution to the gaugino connection can be written as

$$A_\mu = -\frac{x}{3} h^{\nu\rho\sigma} \gamma_{[\mu} \gamma_\nu \gamma_\rho \gamma_{\sigma]}, \tag{27}$$

where $h^{\nu\rho\sigma} = 4x^2\epsilon^{\nu\rho\sigma\mu}\partial_\mu y$ is the 3-form that is dual to the axion in absence of interactions. The axion also appears through the 3-form in a contribution to the gauge boson connection [5]. This suggests that the linear supermultiplet formalism [9] is the natural framework for describing the dilaton supermultiplet, at least in the absence of a superpotential for the dilaton. It has been shown [12] that the axion/3-form duality holds at the quantum level, up to finite topological anomalies. Here we see that when couplings to fermions are included, there are additional anomalies; shifting contributions between T_+ and T_- in (21) is analogous to shifting the integration variable in a Feynman diagram calculation. For example, the linearly divergent triangle diagram leads to an ill-defined finite chiral anomaly that is fixed by imposing invariance under local gauge transformations. In the present case supersymmetry must be used to resolve the ambiguity. With the choice $\zeta = 1$, the gaugino connection (24) is purely "vector-like", and does not contribute to the anomalous $F\tilde{F}$ term that breaks [8] modular invariance, and, by construction [5], is contained in T_-. This agrees with the conclusions of [10], where it was argued that such a contribution is inconsistent with the linearity constraint in the linear multiplet formulation.

To regulate the terms in (26) we introduce chiral supermultiplets $\pi^\alpha = (\bar{\pi}^\alpha)^\dagger$ with

$$K(\pi,\bar{\pi}) = \sum_\alpha (f+\bar{f})|\pi^\alpha|^2, \quad W(\pi) = \sum_\alpha \beta^\alpha_\pi \mu_\pi(z)(\pi^\alpha)^2, \quad \eta^\pi_\alpha = \pm 1. \tag{28}$$

Then

$$\Gamma^\beta_{\alpha i} = \frac{f_i}{2x}\delta^\beta_\alpha, \quad R^\beta_{\alpha i\bar{m}} = -\frac{f_i\bar{f}_{\bar{m}}}{4x^2}\delta^\beta_\alpha, \quad D_\alpha(T_a z)^\alpha = \frac{f_i}{2x}(T_a z)^i = 0. \tag{29}$$

Inserting this into the general expression (7) for $\mathrm{STr}H_\chi$ we get a contribution $\mathrm{STr}H^\pi$ that cancels (26) provided $\sum_\alpha \eta^\pi_\alpha = +N_G$, and the conditions (16) are modified accordingly. Letting $I, J, \cdots$ denote also π_α in (17), we get additional contributions to $\mathrm{STr}(2\mu^2 H' + \nu^2)$:

$$e^{-K}A_{\gamma\beta}\bar{A}^{\alpha\gamma} \;=\; \frac{e^K}{4x^2}\delta^\alpha_\beta|\beta^\pi_\alpha\mu_\pi|^2 \equiv \delta^\alpha_\beta\,(\beta^\pi_\alpha)^2\,\Lambda^2_\pi,$$

$$A_{\alpha\beta i} \quad = \quad \left[K_i - \frac{f_i}{x} - \partial_i \ln \mu_\pi\right] A_{\alpha\beta}. \tag{30}$$

Including these, with $\sum_\alpha \eta^\pi_\alpha \left(\beta^\alpha_\pi\right)^2 = 0$, (20) is modified to include $P = \pi_\alpha,\ \Phi = \pi,\ \zeta_\pi = \zeta'_\pi = 1$.

To fully regulate the theory, including all logarithmic divergences, additional Pauli-Villars fields and/or couplings must be included. Specifically, the superpotential must include the terms

$$W \ni \frac{1}{3} \sum_\alpha W_{ij} Z^I_\alpha Z^J_\alpha, \tag{31}$$

and, to regulate the Yang-Mills contributions, we must include in the set φ^A chiral multiplets $\varphi^A_a = (\bar{\varphi}^A_a)^\dagger$ that transform according to the adjoint representation of the gauge group, with $\sum_{A,a} \eta^a_A = 3N_G$. The field dependence of the corresponding effective cut-off was determined in [10] by imposing the supersymmetric relation between the chiral and conformal anomalies. This in turn determines the Kähler potential: $\alpha^a_A = \frac{1}{3}$. Imposing the full finiteness condition on $\mathrm{STr} H'^2$ may constrain the other parameters $\alpha_A, \alpha_a, \alpha_\alpha$.

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